ECO 244: Applied Mathematics II
Sample Mid-term 1 Exam

1) Let T=$\emptyset(x)$ be the total cost function.
   (a) Write out the expressions for the marginal function M and the average function A.
   (b) Show that, when A reaches a relative extremum, M and A must have the same value.
   (c) What general principle does this suggest for the drawing of a marginal curve and an average curve in the same diagram?

2) Consider a conical drinking cup with height *h* and radius *r* at the open end. The volume of the cup is $V(r,h) = \frac{\pi}{3}r^2h$.
(a) Suppose the cone is now 5 cm high with radius 2 cm. Compute its volume (keep you answer in π)
(b) Compute the partial derivatives $\frac{\partial V}{\partial r}$ and $\frac{\partial V}{\partial h}$ at the current height and radius.
(c) By what amount would the volume change if the cone were lengthened by 10%? (Use the partial derivatives)
(d) If the radius were increased by 5%?

3) A multiproduct monopoly has the following set of price functions:

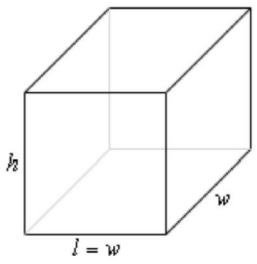
$$P_1 = 55 - Q_1 - Q_2 \qquad and \qquad P_2 = 70 - Q_1 - 2Q_2$$

If the firm's cost function is $(Q) = Q_1^2 + Q_1Q_2 + Q_2^2$, find the profit-maximizing level of output for Product 1 and Product 2.

4) Build a rectangular pen with three parallel partitions using 500 feet of fencing. What dimensions will maximize the total area of the pen?

5) Company ABC is an exports and shipments agency. They ship goods in cardboard boxes made from square sheets, 12 inches by 12 inches. These sheets are cut out in each of the four corners, *x* inches by *x* inches, and folded up to make a box. Find the maximum volume that the box can hold.

6) (HARD) We want to construct a box with a square base and we only have **10 m²** of material to use in construction of the box. Assuming that all the material is used in the construction process, determine the maximum volume that the box can have. (Formula of volume: $V = lwh$)



7) (MODERATE) Jane and Jim invested ***$20,000*** in the design and development of a new product. They can manufacture it for ***$2* per unit**. For the next step, they hired marketing consultants XYZ. XYZ concluded that if Jane and Jim spend ***$a*** on advertising and sell the product at price ***$p*** per unit, they will sell:

$2000 + 4\sqrt{a} - 20p$ units

Using this figure, express the profit that Jane and Jim will make as a function of a and p. What price and level of advertising will maximize their profits? Construct the Hessian determinant to verify your results. **(Note: only price and manufacturing costs are on a per unit basis)**

8) (HARD) The operating cost of a truck is $\mathbf{12 + \frac{x}{6}}$ **cents** per mile when the truck travels at the speed of $\boldsymbol{x}$ **miles per hours**. If the driver earns **$6 per hour (h)**, what is the most economical **speed** to operate the truck on a **400 mile** turnpike? Due to construction, the truck can only travel between **35 and 60 miles per hour**. Verify your answer. (11 points)
Hints*: (I)* $Speed = \frac{Distance}{Time}$ *(II)* $1 = 100\ cents$

9) Find the future value of a principal of $100 at 5% interest rate for 6 years when compounded (a) annually; (b) semiannually; and (c) continually.

10) Cut glass currently worth $100 is appreciating in value according to the formula

$$V = 100e^{\sqrt{t}}$$

How long should the cut glass be kept to maximize its present value under conditions of continuous compounding when (a) r = 0.08 and (b) r = 0.12

11) (MODERATE) The estimated value of a diamond bought for investment purposes is

$$V = 250{,}000(1.75)^{\sqrt[4]{t}}$$

If the discount rate under continuous compounding is 7% how long should the diamond be held? (10 points)