## ECO 173: Applied Statistics <br> Sample Midterm 1 Exam

This sample exam does not cover all the materials you are responsible to know for your midterm 1 exam. This sample is designed to give you an idea of the question pattern and the level of difficulty you are likely to face in the exam. You are advised to peruse the textbook chapters, class notes, slides and worksheets to be fully prepared for this exam.

1. The GPA of college students is normally distributed with a mean of 3.00 and a standard deviation of 0.25 . The 65th percentile of GPA is*
(a) 0.6517
(b) 0.39
(c) 0.0975
(d) 3.0975

* Hint: the value of the $z$-statistic is such that 0.65 probability lies to the left.

2) A type 1 error is:
a) Failing to reject the null hypothesis when the alternative hypothesis is true.
b) Failing to reject the null hypothesis when the alternative hypothesis is false.
c) Rejecting a true null hypothesis.
d) Rejecting a false null hypothesis.
3) Given a P-value of 0.07 , we:
a) Reject the null hypothesis for $\alpha=.01$
b) Reject the null hypothesis for $\alpha=.05$
c) Fail to reject the null hypothesis for $\alpha=.1$
d) Fail to reject the null hypothesis for $\alpha=.05$
4) At a supermarket, the average number of register mistakes per day was 18. The owner of the supermarket purchased new cash registers in an effort to decrease the number of errors. A random sample of 30 days was collected to determine whether the mean number of mistakes per day, $\mu$, is lower with the new registers. What would be the most appropriate null and alternative hypotheses?
(a) $\mathrm{H}_{0}: \mu=18, \mathrm{H}_{1}: \mu \neq 18$
(b) $H_{0}: \mu \geq 18, H_{1}: \mu<18$
(c) $\mathrm{H}_{0}: \mu<18, \mathrm{H}_{1}: \mu \geq 18$
(d) $\mathrm{H}_{0}: \mu>18, \mathrm{H}_{1}: \mu \leq 18$
5) Consider the population of all students, undergraduate and graduate, attending the University of Kentucky. You are told that the average age is 21.6 and that the standard deviation is 1.4 years.

Given this information, what is the mean of the sampling distribution of sample mean when the sample size is 50 ?
(a) 50
(b) 1.4 .
(c) $1.4 / \mathrm{V} 50$
(d) 21.6
6) If you did not reject a null hypothesis at the $5 \%$ level of significance, you would definitely reject it at $1 \%$.
(a) TRUE
(b) FALSE
3. True/False: Suppose that a random sample of size $\mathrm{n}=3$ is drawn from a population that has mean 7 and variance 9 . The sampling distribution of $\bar{X}$ will have mean $=7$ and variance $=3$. Is this statement true? Explain.
4. Battery lifetime is normally distributed for large samples. The mean lifetime is 500 days and the standard deviation is 61 days. To the nearest percent, what percent of batteries have lifetimes longer than 561 days?
5. Professor Halen has 184 students in his college mathematics lecture class. The scores on the midterm exam are normally distributed with a mean of 72.3 and a standard deviation of 8.9. How many students in the class can be expected to receive a score between 82 and 90 ? Express answer to the nearest student.
6. A group of 625 students has a mean age of 15.8 years with a standard deviation of 0.6 years. The ages are normally distributed. How many students are younger than 16.2 years? Express answer to the nearest student?
7. The cost of treatment per patient for a certain medical problem was modeled by one insurance company as a normal distribution with mean $\$ 775$ and standard deviation $\$ 150$.
(a) What is the probability that the treatment cost for a randomly chosen patient is more than $\$ 800$ ?
(b) What is the probability that the total treatment cost for 15 randomly chosen patients is more than \$12,000? (Hint: convert to average)
(c) What is the probability that the average treatment cost for 35 randomly chosen patients is less than $\$ 750$ ?
(d) The probability that 20 randomly chosen people have a treatment cost above a certain total is 0.23 . What is the total value?
(e) The probability that 40 randomly chosen people have an average treatment cost less than a certain average is 0.18 . What is the average value?

For \#20-25: Use Table A (Standard Normal Distribution) to find the proportion of observations that satisfies each of the following statements. In each case, shade the area under the curve that is the answer to the question.


For \#26-28: Find the value of $z$ from the standard normal distribution that satisfies each of the following conditions. (Use the value of $z$ from Table $A$ that comes closest to satisfying the condition.)
26.


$z=$ $\qquad$

$z=$ $\qquad$
8. A university wants to know more about the knowledge of students regarding international events. They are concerned that their students are uninformed in regards to news from other countries. A standardized test is used to assess student's knowledge of world events (national reported mean = 65). A sample of 30 students are tested (sample mean $=58$ ). Assuming the knowledge of students in the university has a standard deviation of 5, compute a 99 percent confidence interval based on this sample's data. How do these students compare to the national average?
9. Consider a population with $\mu_{y}=100$ and $\sigma^{2}=43$ (note that this is the variance rather than the standard deviation).
a. For a random sample of size 100 , find $\operatorname{Pr}(Y$-bar <101) .
b. For a random sample of size 64, find $\operatorname{Pr}(101<Y$-bar $<03)$.
c. For a random sample of size 165 , find $\operatorname{Pr}(\mathrm{Y}$-bar $>98)$.

Hint: Apply Central Limit Theorem
10. Scores on a Wechsler Adult Intelligence Scale, a standard IQ test, are approximately normal for 20 to 34 age group with $\mu=110$ and $\sigma=25$.
(a) What percent of this age group have an IQ less than 100 ? (Percentage is simply Probability X 100 )
(b) What percent of this age group have an IQ between 90 and 115?
(c) Find the $80^{\text {th }}$ percentile of the IQ scores distribution.
(d) Find the IQ scores which separates the lowest $25 \%$ of all IQ scores for this age group.
11. Suppose that we randomly select a sample of 64 measurements from a population having a mean equal to 20 and standard deviation equal to 4.
(a) Describe the shape of the sampling distribution of the sample mean $\bar{x}$. Do we need to make any assumptions about the shape of the population? Why or why not?
(b) Find the mean and the standard deviation of the sampling distribution of the sample mean.
(c) Calculate the probability that a single measurement will be greater than 21.
(d) Calculate the probability that we will obtain a sample mean greater than 21.
12. The average length of a hospital stay in the US is $\mu=9$ days with standard deviation of $\sigma=3$ days. Assume a simple random sample of 100 patients is obtained and the mean stay for 100 patients is obtained. What is the probability that the average length of stay for this group of patients will be less than 9.6 days?

For Problems 13, 14 and 15 follow the given steps to conduct a comprehensive hypothesis test. You are expected to memorize these steps, and apply them in the exact order in your exam for z-tests.

Step 1: State the theory/question you are about to test.
Step 2: State the test design (i) Null and Alternative Hypothesis (ii) Right, Left or Two tailed test (iii) Distribution of the sampling distribution of sample mean

## Step 3: Compute the test statistic

Step 4: Find the critical values of the rejection region. Include sketches of the sampling distribution $\bar{X}$ and the standardized sampling distribution $Z_{\bar{X}}$ to show the rejection regions.

Step 5: State your decision. Interpret your decision with respect to the theory stated in Step 1.

## Step 6: Calculate the p-value.

13. The life span of 100 W light bulbs manufactured by a particular company follows a normal distribution with a standard deviation of 120 hours and its mean half-life is guaranteed under warranty for a minimum of 800 hours. At random, a sample of 50 bulbs from a lot is selected and it is revealed that the mean half-life is 750 hours. With a significance level of 0.01 , should the lot be rejected by not honoring the warranty?
14. The college bookstore tells prospective students that the average cost of its textbooks is $\$ 52$ with a standard deviation of $\$ 4.50$. A group of smart statistics students thinks that the average cost is higher. In order to test the bookstore's claim against their alternative, the students will select a random sample size of 100 . Assume that the mean from their random sample is $\$ 52.80$. Test the theory at $5 \%$ level of significance.
15. It is believed that the average level of prothrombin in a normal population is 20 $\mathrm{mg} / 100 \mathrm{ml}$ of blood plasma, with a standard deviation of $4 \mathrm{mg} / 100 \mathrm{ml}$. To verify this, a sample is taken from 40 individuals in whom the average is $18.5 \mathrm{mg} / 100 \mathrm{ml}$. Is there enough statistical evidence present to infer that the population average level of prothrombin in 100 ml of blood plasma is different from 20 mg ?

Questions from external worksheets:
3. The mean annual salary for registered nurses in the U.S. is $\$ \mathbf{3 8}, \mathbf{0 0 0}$. Assume that $\sigma=\$ 1700$.
a. A sample of 15 nurses selected. What is the probability that their mean annual salary is less than $\$ 34,500$ ?
b. A sample of 65 nurses selected. What is the probability that their mean annual salary is more than $\$ 38,600$ ?

You work for a consumer advocate agency and want to find the mean repair cost of a washing machine. As part of your study, you randomly select 40 repair costs and find the mean to be $\$ 100.00$. From past studies, you assume that the $\sigma$ is $\$ 17.50$.
a. Construct a $90 \%$ confidence interval.
b. Construct a $95 \%$ confidence interval.
c. Construct a $99 \%$ confidence interval.

Interpret your answer for each.

