1. Firm A's cost function is $C(x_1, x_2) = 3x_1^2 + 6x_1 + 6x_2^2$, where x_1 and x_2 denote the quantities of good 1 and good 2 produced. The firm has to meet a combined production target of 50 units. Find the values of x_1 and x_2 that will minimize the cost using Lagrange Multiplier Method. Verify your results using a Bordered Hessian.

2. A consumer wants to maximize his/her utility. The consumer's utility function is given by $U(x1, x2) = 2x1x2 + 3x1$ where $x1$ and $x2$ denote the number of good 1 and good 2 purchased. The budget constraint is given as $P_{x1}x_1 + P_{x2}x_2 = 83$ where the price is $P_{x1} = \$1$ and $P_{x2} = \$2$.

(a) Form the lagrangian function and find the critical values of x1, and x2

(b) Using the bordered Hessian matrix, check if the utility is maximized subject to the budget constraint.

3. Evaluate the following integrals:

(a) $\int lnx \ dx$ (use Integration by parts, where v = ln x)

(b) $\int x^3 lnx \ dx$ (use Integration by parts)

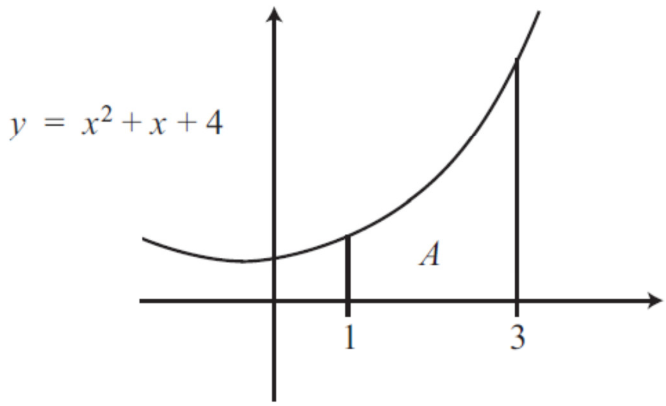
(c) $\int \frac{e^x}{1+e^x} \ dx$ (use substitution)

(d) $\int_0^3 (x+3)(x+2)^{-4} \, dx$ (use integration by parts)

(e) $\int_0^1 x\sqrt{1-x} \ dx$ (use integration by parts)

(f) $\int \frac{6x}{x^2+1} \ dx$ (use substitution)

(g) $\int x(x^2+1)^{10} dx$ (use substitution)



4. Find Area A.

5. Minimize Firm A's total costs $c = 45x^2 + 90xy + 90y^2$ when the firm has to meet a production quota g equal to $2x + 3y = 60$. Use Lagrangian method to find critical values and verify your answer by Bordered Hessian.

6. Consider the Cobb-Douglas production function: $Q = 20K^{1/2}L^{1/2}$
   (a) Prove that the production function is linearly homogenous.
   (b) Prove the three properties of linearly homogenous production function for the given Cobb-Douglas function.
7. Consider the Cobb-Douglas production function: $Q = 20K^{3/4}L^{1/4}$
   (a) Prove that the production function is linearly homogenous.
   (b) Prove the three properties of linearly homogenous production function for the given Cobb-Douglas function.
8. Consider the following production function:
$$Q = \frac{K^2 - L^2}{L}$$
   (a) Show that the function is linearly homogenous.
   (b) Prove the three properties of linearly homogenous production functions.

9. Consider the following production function:
$$Q = K + \frac{L^3}{K^2}$$
   (c) Show that the function is linearly homogenous.
   (d) Prove the three properties of linearly homogenous production functions.