BUS 135 Applied Mathematics

Sample Midterm 1 Exam

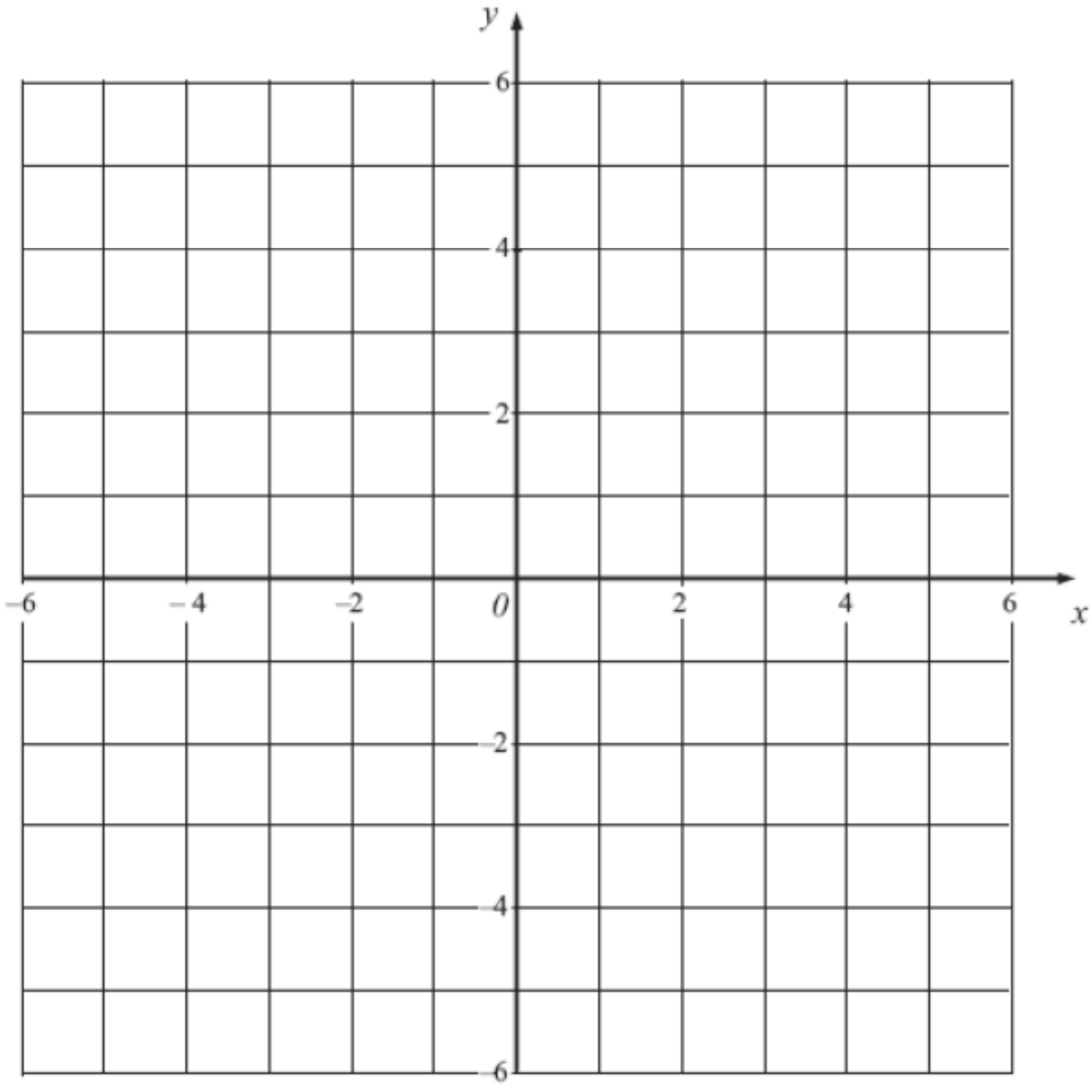
A. Sketch the following functions and inequalities:

1. $y = 4$
2. $x + y = -7$
3. $2x + y = -1$
4. $-4x = 5 + y$
5. $y = -2x - 4$
6. $3y = -24$
7. $2x + 4y = -3$
8. $5y = 8$
9. $-6y = 8 + 4x$
10. $y = \frac{3}{11}x + \frac{10}{11}$
11. $y = -x - 1$
12. $y = -x - \frac{1}{2}$
13. $y \geq 4x - 4$
14. $y \leq \frac{5}{2}x + 2$
15. $y \geq -\frac{7}{5}x - 3$
16. $y \geq -\frac{3}{4}x + 4$

B. Multiple inequalities sketching

1. On the grid, shade the region that satisfies all three of these inequalities

$y > -4$ $\qquad$ $x < 2$ $\qquad$ $y < 2x + 1$

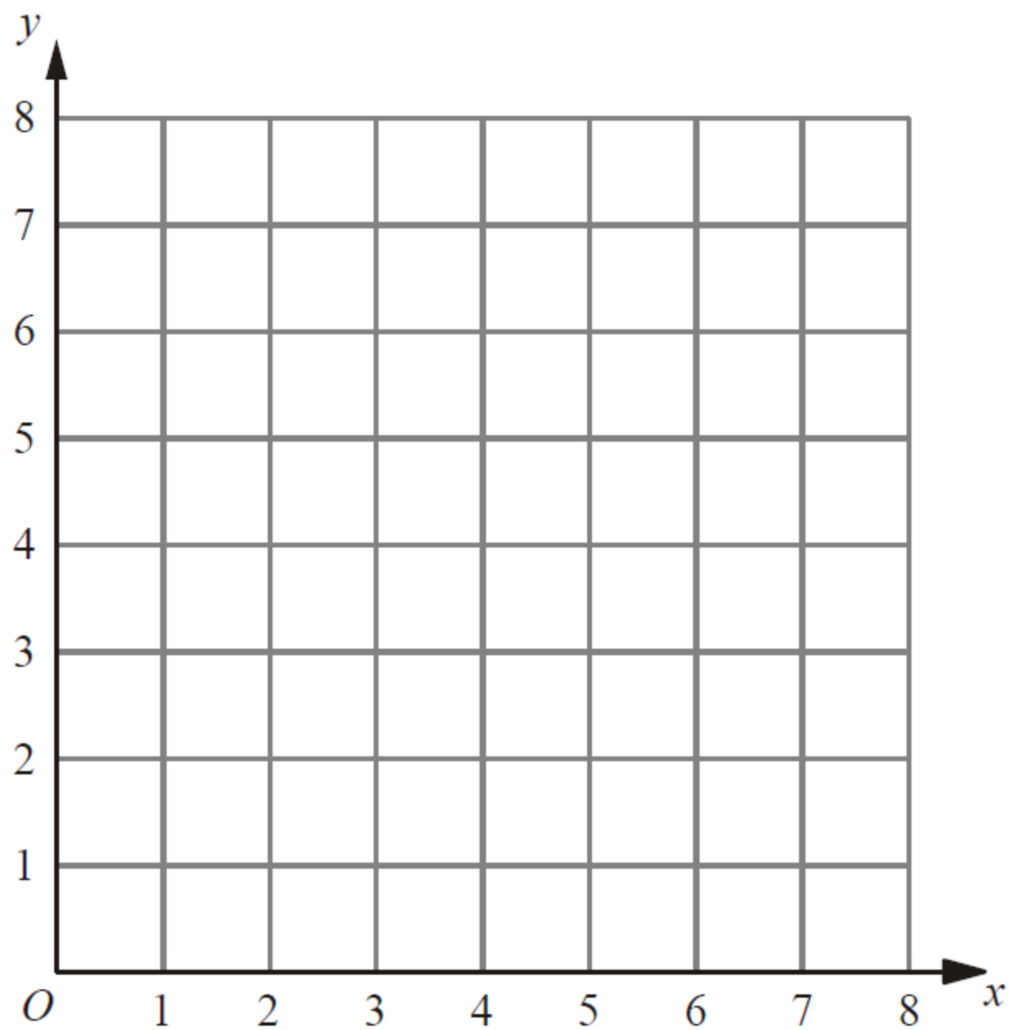


NA

2. The region **R** satisfies the inequalities

$$x \geq 2, \quad y \geq 1, \quad x + y \leq 6$$

On the grid below, draw straight lines and use shading to show the region **R**.



Use Cramer's Rule to solve each system.

1) $x - 5y = -5$
$-4x - 2y = 20$

2) $-x + 5y = 2$
$x - 2y = -2$

3) $2x + 2y = 0$
$4x - y = -20$

4) $3x - 4y = 1$
$-5x + 2y = 3$

NA

5) $-x - y = -1$
$\quad 3x + 3y = 3$

6) $-5x + 5y = 10$
$\quad -2x + 2y = -4$

D. Linear Programming

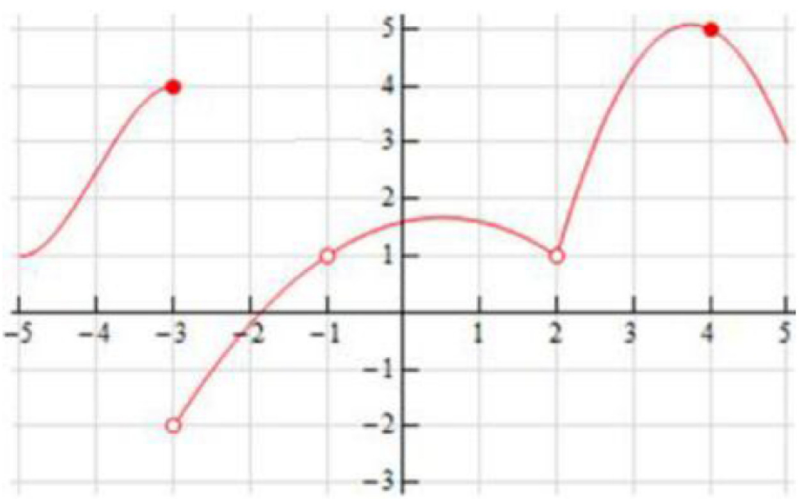
*Practice the problems recommended from the textbook first, and then attempt to solve this problem:

$Maximize\ z = (x - 45) + (y - 5)\ subject\ to\ the\ following\ contraints\ (i)\ 50x + 24y\ \leq 2400\ \ (ii) 30x + 33y\ \leq 2100\ \ (iii)\ x \geq 45\ \ and\ (iv)\ y \geq 5$

E. Limit and Continuity

Below is the graph of $f(x)$. For each of the given points, clearly state if the limit $\lim_{x \to a} f(x)$ exists and if the function is continuous. (The circles that are filled indicate there is **no break/gap** at that point. The circles that are **hollow** indicate **breaks/gaps** at those points)

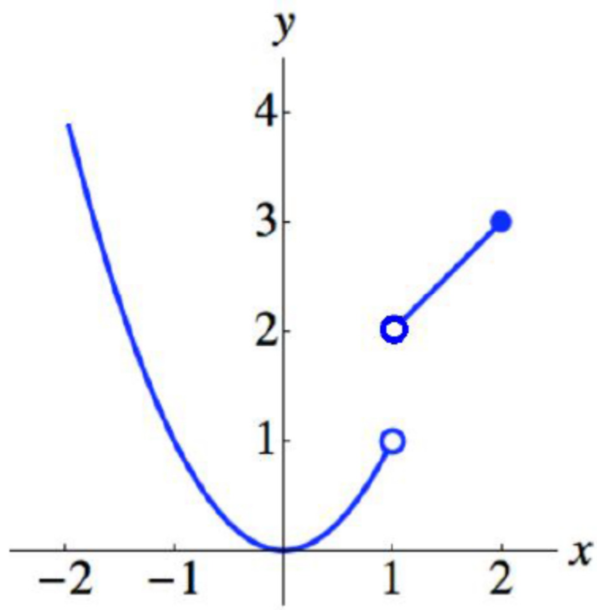
(a) $a = -3$ **(b)** $a = -1$ **(c)** $a = 2$ **(d)** $a = 4$



Consider the following graph below:

* The hollow circle indicates there is a gap/break in the function at that point.

A) Is the function continuous at $= -1$?
B) Is the function continuous at $= 1$?



Evaluate the following limits:

3. $\lim_{x \to 2} x(x-1)(x+1)$

4. $\lim_{x \to 3} x^3 - 3x^2 + 9x$

9. $\lim_{x \to -1} \dfrac{x^2+6x+5}{x^2-3x-4}$

10. $\lim_{x \to 2} \dfrac{x^2-4x+4}{x^2+x-6}$

11. $\lim_{x \to -1} \dfrac{2x^2+x-1}{x+1}$

12. $\lim_{x \to 1} \dfrac{3x^2-x-2}{2x^2+x-3}$

5. $\lim_{x \to 3} \dfrac{x^2-2x}{x+1}$

7. $\lim_{x \to 1^+} \dfrac{x^4-1}{x-1}$

23. $\lim_{y \to 6} \dfrac{y+6}{y^2-36}$

26. $\lim_{x \to 4} \dfrac{3-x}{x^2-2x-8}$

29. $\lim_{x \to 9} \dfrac{x-9}{\sqrt{x}-3}$

30. $\lim_{y \to 4} \dfrac{4-y}{2-\sqrt{y}}$

F. Differentiation

NA

Q1. Find $\frac{dy}{dx}$ for the following:

a) $y = \frac{1}{x}$ b) $y = \frac{1}{x+1}$ c) $y = x^2 - x$

d) $y = x^4$ e) $y = \frac{1}{\sqrt{x}}$ f) $y = \frac{1}{\sqrt{x-1}}$

g) $y = 2x^2$ h) $y = \frac{1}{x^2}$ i) $y = x^3$

j) $y = 2x^3 + 1$ k) $y = \sqrt{x+1}$ l) $y = \sqrt{2x^3 + 1}$

Find $f'(x)$ of the following:

a) $f(x) = x^3 + 5$

b) $f(x) = x^2 (x^3 + 5)$

c) $f(x) = \frac{x^3 + 5}{2}$

d) $f(x) = \frac{x^3 + 5}{x^2}$

e) $f(x) = x^{-3} + \frac{1}{x^7}$

f) $f(x) = \sqrt{x} + \frac{1}{x}$

g) $f(x) = \sqrt[3]{\frac{8}{x}}$

h) $f(x) = \frac{x^{3/2} + 2}{x}$

Find $\frac{dy}{dx}$ of the following:

a) $y = 1 + x + x^2 + x^3 + x^4 + x^5$

b) $y = \frac{1 + x + x^2 + x^3 + x^4 + x^5 + x^6}{x^3}$

c) $y = (1-x)(1+x)(1+x^2)(1+x^4)$

d) $y = x^{24} + 2x^{12} + 3x^8 + 4x^6$

NA

Q9. Use the Product Rule or Quotient Rule for the following

a) $f(x) = (3x^2+6)\cdot(2x-\frac{1}{4})$

b) $f(x) = (2-x-3x^3)\cdot(7+x^5)$

c) $f(x) = (x^3+7x^2-8)(2x^{-3}+x^{-4})$

d) $f(x) = (\frac{1}{x}+\frac{1}{x^2})\cdot(3x^3+27)$

e) $f(x) = \dfrac{3x+4}{x^2+1}$

f) $f(x) = \dfrac{x-2}{x^4+x+1}$

g) $f(x) = \dfrac{x^2}{3x-4}$

h) $f(x) = \dfrac{2x^2+5}{3x-4}$

Q10. Use the Chain Rule for the following: [Find $\frac{dy}{dx}$]

a) $y = (x^7+2x-3)^3$ $\quad 3u^2 \cdot \frac{du}{dx}$ ✓

b) $y = (x^2+1)^4$

c) $y = (x^3+2x)^{37}$

d) $y = (3x^2+2x-1)^6$

e) $y = (x^3-\frac{7}{x})^{-2}$

f) $y = \dfrac{1}{(x^5-x+1)^9}$

g) $y = \dfrac{4}{(3x^2-2x+1)^3}$

h) $y = \sqrt{x^3-2x+5}$

i) $y = \left(\dfrac{x-5}{2x+1}\right)^3$

✓ j) $y = \left(\dfrac{1+x^2}{1-x^2}\right)^{17}$

k) $y = \dfrac{(2x+3)^3}{(4x^2-1)^8}$

l) $y = x^2\sqrt{5-x^2}$

Find the partial derivatives of the following equations:

Q3. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$:

a) $z = \dfrac{xy}{x^2+y^2}$

b) $z = \dfrac{x^2y^2}{x+y}$

Q4. Find $f_x(x,y)$ and $f_y(x,y)$:

a) $f(x,y) = 3x^4y - 7x^3y$

b) $f(x,y) = \dfrac{x+y}{x-y}$

Q5. Evaluate the indicated partial derivative:

$$f(x,y) = 9 - x^2 - 7y^3; \quad f_x(4,1) \text{ and } f_y(4,1)$$

Q6. Let $f(x,y,z) = x^2y^4z^3 + xy + z^2 + 1$. find:

a) $f_x(x,y,z)$ b) $f_y(x,y,z)$ c) $f_z(x,y,z)$

NA