1. Optimize the following functions. Use second order conditions to determine whether the function reaches a maximum/minimum/inflection point at the critical values.

(a) $z = f(x, y) = 60x + 34y - 4xy - 6x^2 - 3y^2 + 5$

(b) $z = f(x, y) = 48y - 3x^2 - 6xy - 2y^2 + 72x$

(c) $z = f(x, y) = 5x^2 - 3y^2 - 30x + 7y + 4xy$

2. Optimize the following functions. Use Hessian matrix to determine whether the function reaches a maximum/minimum/inflection point at the critical values.

(a) $Z = f(x_1, x_2, x_3) = x_1^2 + (x_1 + x_2)^2 + (x_1 + x_3)^2$

(b) $Z = f(x_1, x_2, x_3) = -x_1^2 - 3x_2^3 - 10x_3^2 + 4x_1 + 24x_2 + 20x_3$

(c) $Z = f(x_1, x_2, x_3) = x_1x_2 + x_2x_3 + x_3x_1 - 2x_1 - 2x_2 - 2x_3$

3. Optimize the following functions subject to the given constraint. Use Bordered Hessian matrix to determine whether the function reaches a constrained maximum / constrained minimum.

(a) $z = f(x, y) = 4x^2 - 2xy + 6y^2$ subject to $x + y = 72$

(b) $z = f(x, y) = 26x - 3x^2 + 5xy - 6y^2 + 12y$ subject to $3x + y = 170$