

ECO 134: Applied Mathematics I

Chapter 2: Relations and Functions

A set containing elements $\{a,b\}$ can also be written as $\{b,a\}$, i.e. $\{a,b\} = \{b,a\}$. Hence the order of elements in the set does not matter. In this case, a and b are an unordered pair.

However, there might be cases where the ordered pair carries significance. We can write two different ordered (a,b) and (b,a) which have the property $(a,b) \neq (b,a)$, unless $a = b$. Sets with brackets like these “()” are known as ordered pairs, e.g. x and y coordinate points.

Example: A kindergarten teacher records the age (years) and weight (pounds) of each student in her class as an ordered set, where the first element of the ordered set indicates age and the second indicates weight. For example, a student aged 4 weighs 40 pounds, so the set is $(4, 40)$. Obviously if the set was written as $(40, 4)$ it would be a major error! Hence, order in this case is very important.

Cartesian coordinate system: In the rectangular / Cartesian coordinate plane – x -axis and y -axis cross each other at right angles, thus dividing the plane in four quadrants – the first element represents the x -value and the second element represents the y -value.

Relation vs. Function

Consider the two sets:

$$\text{Set A } \{(x,y) \mid y = 2x\}$$

$$\text{Set B } \{(x,y) \mid y \leq x\}$$

Both Set A and Set B depict a relationship between x and y , so Sets A and B are known as relations. Set A is a special type of relation known as function.

In a relation, each value of x **may** correspond to one or more than one value of y . Since $y \leq x$ in set B, then set B contains coordinate points like $(1,0)$, $(1,1)$, $(2, - 4)$ etc.

A function is a special kind of relation where each value of x corresponds to only one (i.e. unique) value of y . Hence Set A is a function. More formally, a function is a set of ordered pairs with the property that any x value uniquely determines a y value.

Characteristics of a function:

- 1) A function is always a relation, but a relation is not always a function.
- 2) A function requires there to be a unique y for each x , however it is not necessary for there to be a unique x for each y , i.e. more than one value of x may be associated with each value of y , e.g. $y = 6$.
- 3) A function is also a mapping or transformation. A function can be written as this $y = f(x)$, where f can be interpreted as the way by which the set of x -values are mapped on the set of y -values, i.e. how x is translated into y .
- 4) A function is usually written as $y = f(x)$. However, it can also use other letter like $y = g(x)$ or $y = h(x)$. For instance quantity demanded function can be written as $Q_d = D(P)$, where D is the same thing as f .

Components of a function

- x is known as the argument of a function, or the independent variable.
- y is known as the value of a function, or the dependent variable.
- The set of all permissible values / the set of values that x can take is known as the domain of a function.
- The set of all permissible values / the set of values that y can take is known as the range of a function.