Matrix Algebra

Sec 4.5 and 4.6 Theoretical Handout

Identity Matrix: An identity matric is a square matrix that has 1s along the principal diagonal and 0s everywhere else. The symbol for identity matrix is $I$.

Properties of Identity Matrix:

* Identity matrix has a special property that $IA=AI=A$.
* Inserting an identity matrix between two matrix will not change the product, $AIB=AB$
* Squaring an identity matric is equal to the identity matrix

$$I^{2}=I×I=I$$

Any matrix that remains unchanged when multiplied to itself is said to be idempotent.

Null Matrix: A null matrix is simply a matrix of 0s. A null matrix may be square or non-square.

Transpose

Consider a matrix A. If the columns and rows of matrix A are interchanged, so that its first row becomes its first column, or vice-versa, then we obtain the transpose of A. The transpose of A is given as $A^{'}$ or $A^{T}$.

Properties of Transpose Matrix
1. $\left(A^{T}\right)^{T}=A$ : This means the transpose of a transpose is the original function

2. $\left(A+B\right)^{T}=A^{T}+B^{T}$

3. $\left(AB\right)^{T}=B^{T}A^{T}$

Inverse Matrix

Consider matrix A. The inverse of A is $A^{-1}$.

Properties of Inverse matrix:

* $AA^{-1}=A^{-1}A=I$
* Inverses can only exist for square matrices, but not all square matrices have inverses.
* If a square matrix has an inverse, it is called nonsingular
* If a matrix (square or non-square) does not have an inverse, it is called singular
* If the inverse of $A$ is $A^{-1}$, then the inverse of $A^{-1} $is $A$
* If $A$ has the dimension $\left(n×n\right) $then $A^{-1}$ is also of the dimension $\left(n×n\right)$
* The inverse of a nonsingular matrix is unique
* $\left(A^{-1}\right)^{-1}=A$ : inverse of an inverse is the original matrix
* $\left(AB\right)^{-1}=B^{-1}A^{-1}$
* $\left(A^{T}\right)^{-1}=\left(A^{-1}\right)^{T}$