Hypothesis Testing for Population Mean $\left(μ\right)$

Case 1:

* Population mean is unknown
* Population standard deviation is known
* Sample size < 30
* Population is normally distributed

In this case, we can use normal distribution to run the hypothesis test

Case 2:

* Population mean is unknown
* Population standard deviation is known
* Sample size >= 30
* Population may or may not be normally distributed

In this case, we can use normal distribution to run the hypothesis test

Case 3:

* Population mean is unknown
* Population standard deviation is known
* Sample size < 30
* Population is not normally distributed

In this case, use non-parametric method.

Case 4:

* Population mean is unknown
* Population standard deviation is unknown
* Sample size < 30
* Population is normally distributed

In this case, use t-distribution to perform hypothesis test.

Case 5:

* Population mean is unknown
* Population standard deviation is unknown
* Sample size if >= 30
* Population may or may not be normally distributed

In this case, use t-distribution to perform hypothesis test

Case 6:

* Population mean is unknown
* Population standard deviation is unknown
* Sample size if < 30
* Population is not normally distributed

In this case, use non-parametric method.

In Chapter 9, we only work with cases 1, 2, 4, 5.

**Methods to run hypothesis test**

There are two ways to run a hypothesis test.

1. Rejection region approach
2. P-value approach

For **Cases 1 and 2**, we use normal distribution to run the hypothesis test.

Suppose sample mean $=\overbar{X}$. We first have to standardize this value:

$$Z\_{\overbar{X}}=\frac{\overbar{X}-μ}{^{σ}/\_{\sqrt{n}}}$$

Significance level: $α $(usually very small, like 0.1, 0.05, 0.025..)

**Left-tailed test**

* Rejection Region Approach
1. Find the z-critical value that has an area of $α$ to the **left**.
2. Compare $Z\_{\overbar{X}}$ to the z-critical value



Rejection Rule for left tailed test:

Reject $H\_{0}$ if $Z\_{\overbar{X}}\leq $ z critical value

Do not reject $H\_{0}$ if $Z\_{\overbar{X}}>$ z critical value

* P-value Approach

For left tailed test, find the area to the left of $Z\_{\overbar{X}}$



Reject $H\_{0}$ if p-value < $α$

Do not reject $H\_{0}$ if p-value > $α$

**Right-tailed test**

* Rejection Region Approach
1. Find the z-critical value that has an area of $α$ to the **right**
2. Compare $Z\_{\overbar{X}}$ to the z-critical value



Rejection Rule for right tailed test:

Reject $H\_{0}$ if $Z\_{\overbar{X}} \geq $ z critical value

Do not reject $H\_{0}$ if $Z\_{\overbar{X}}<$ z critical value

* P-value Approach

For a right-tailed test, p-value is the area to the right of $Z\_{\overbar{X}}$.

Reject $H\_{0}$ if p-value < $α$

Do not reject $H\_{0}$ if p-value > $α$



**Two-tailed test**

* Rejection Region Approach
1. Find two z-critical values. One that has an area of $α/2$ to the **right**, and one that has an area of $α/2$ to the **left.**
2. Compare $Z\_{\overbar{X}}$ to the z-critical values



Rejection Rule:

Reject $H\_{0}$ if $Z\_{\overbar{X}}\leq z\_{1}$ or if $Z\_{\overbar{X}}\geq z\_{2}$

Do not reject $H\_{0}$ if $z\_{1}<Z\_{\overbar{X}}<z\_{2}$

* P-value Approach

For two-tailed tests, p-value is calculated as follows:

If $Z\_{\overbar{X}}<0$ , then p-value is $2×P\left(Z<Z\_{\overbar{X}}\right)$

If $Z\_{\overbar{X}}>0$ , then p-value is $2×P\left(Z>Z\_{\overbar{X}}\right)$

Reject $H\_{0}$ if p-value < $α$

Do not reject $H\_{0}$ if p-value > $α$

Handout for hypothesis tests for Cases 4 and 5 will be distributed next class.